From the measurement to the result

The measurement

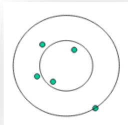
No experimental measurement technique is rigorously reliable. Every measurement is subject to error: there is no such thing as an exact measurement. The “true” value of a measurement is technically inaccessible.
An experimental result must therefore always be given taking this error into account.

1. *Sources of error*

a. *Random error*

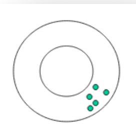
An error is said to be random when, by carrying out a large number of measurements of the same quantity under the same conditions, we find that the different results are distributed on either side of the mean value obtained.
The origin of a random error is generally linked to the inaccuracy of the measuring device (discrete graduations) and to the “lack of rigor” of the experimenter (this contribution is, however, difficult to quantify).

Note: Sometimes, the random error may be linked to the measured phenomenon itself, if it is particularly unstable. Weather forecasts, for example, are based on highly unstable data subject to considerable variability.



When an experiment is subject to random error, the results of repeated measurements under the same conditions are far apart. However, this does not prevent the mean value from being correct.

b. *Systematic error*



An error is said to be systematic when it always takes the same value on each repeated measurement. It therefore always affects the result in the same direction.
The origin of a systematic error is generally a defect in the measuring device or in the experimental protocol. It is difficult to see on the raw data, but can sometimes be easily corrected during data analysis.
When an experiment is marred by a systematic error, the results of repeated measurements under the same conditions are very close. The instrument used is therefore accurate. However, the measured value is not accurate.

2. *Precision vs. Accuracy*

Let G be the measured quantity.

a. *Standard uncertainty*

The standard uncertainty, or measurement uncertainty, is a parameter associated with the measurement result, enabling its quality to be judged.
Noted U(G), the standard uncertainty is a number of the same unit as the measurement performed. It is always written with a single non-zero digit.
The experimental result should then be written: $\boldsymbol{G = G_{measured} + U(G)}$

b. *Some usual uncertainties*

Generally speaking, the standard uncertainty of an instrument corresponds to half its gradation.
When using chemistry equipment, standard uncertainties are generally supplied by the manufacturer and indicated on the equipment.

Ex: Let us measure length l_1 = 15.2 cm with a ruler graduated in mm.
$U(l_1)$ = 0.5 mm, and the result of the measurement will be written: $l = 152 \pm 0.5\ mm$
If the ruler is graduated in half-mm, $U(l_2)$ = 0.25 mm, and $l = 152 \pm 0.3\ mm.$ *(Always only 1 digit for uncertainty)*
Let us measure a length l_3 = 384000 km with a stick of 1 m length: $l = 384000000 \pm 0.5\ m$

c. Relative uncertainty

To determine the accuracy of a measurement, the standard uncertainty is no longer sufficient. We need to calculate its relative uncertainty, $\frac{\Delta G}{G} = \frac{U(G)}{G}$. This is a unitless number, often expressed as a percentage.

Ex: $\left(\frac{\Delta l}{l}\right)_1 = \frac{0.5}{152} = 3.3 \times 10^{-3}$

$\left(\frac{\Delta l}{l}\right)_2 = \frac{0.25}{152} = 1.6 \times 10^{-3}$

The second measurement is more accurate than the first one.

$\left(\frac{\Delta l}{l}\right)_3 = \frac{0.5}{384000000} = 1.3 \times 10^{-9}$

Even if the 3rd instrument is less precise than the first 2 ($U(l_3) > U(l_1) > U(l_2)$), the 3rd measurement is more accurate than first 2.

Writing a number in sciences

In natural sciences, a measurement and/or a result is never exact, but only corresponds to an interval of greater or lesser precision.

1. Significant digits/figures

This precision can be expressed through the use of significant digits (SD). The writing of a number reflects the precision with which it has been measured and/or calculated. This is achieved through the use of significant figures (SF). This is the set of digits, starting with the 1st non-zero digit, with which the number is written

Ex: *1.56 is written with 3 SF.* *1.560 is written with 4 SF.*
15.6 is written with 3 SF. *15.60 is written with 4 SF.*
0.0156 is written with 3 SF. *0.01560 is written with 4 SF.*

Note: *When in doubt, simply switch to scientific notation to find out which digits are significant and which are not.*
1.560 remains 1.560 ; 15.60 becomes 1,560x10^{1} ; 0.01560 becomes 1,560x10^{-2} ;
0,0156 becomes 1,56x10^{-2} (The first 2 "0" disappear)

2. Significant figures and precision

1.56 corresponds to the set of numbers that can be rounded to this value, i.e. all numbers in the interval $[1.555 ; 1.565[$. *It can be written* 1.56 ± 0.005.

The relative uncertainty on this number is $\frac{0.005}{1.56} = 3 \times 10^{-3}$

1.560 corresponds to the set of numbers that can be rounded to this value, i.e. all numbers in the intervall $[1.5695 ; 1.5605[$. *It can be written* 1.560 ± 0.0005.

The relative uncertainty on this number is $\frac{0.0005}{1.560} = 3 \times 10^{-4} < 3 \times 10^{-3}$

The greater the number of significant figures, the greater the precision.

3. Significant figures and calculations

The vast majority of calculations in the sciences involve multiplication and division. We can therefore define a 1st simple rule for taking uncertainties into account when calculating using measurements:
When multiplying and/or dividing measured values, the final result is written with as many significant digits as the least precise measurement, i.e. the one written with the fewest significant digits.

Ex:
$15.6 \times 1.6 = 25$ *The calculator gives 24.96, but only 2 SF*
$15.6 \times 1.56 = 24.3$ *The calculator gives 24.336, but only 3 SF*
$15.6 \times 1.560 = 24.3$ *The calculator gives 24.336, but only 3 SF*
$15.60 \times 1.560 = 24.34$ *The calculator gives 24.336, but only 4 SF*
$15.6000 \times 1.56000 = 24.3360$ *The calculator gives 24.336, but 6 SF*

Note: *Round off, not truncate.*

Ex: $\frac{1}{3} = 0.3 ; \frac{1.0}{3.0} = 0.33 ; \frac{1.00}{3.00} = 0.333 ; \frac{1.0}{3.00} = 0.33$